# Conceptual change in mathematics: Learning about fractions may provoke changes in children's prior whole number knowledge


Pooja G. Sidney & Martha W. Alibali

University of Wisconsin - Madison

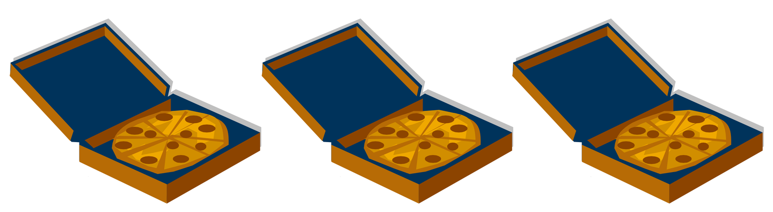

## Fractions and Whole Numbers: Two Incommensurable Systems?

**Fractions Are a Difficult Domain**

- True in part because of the whole number bias (Ni & Zhou, 2005).
- Children have well-practiced knowledge of whole numbers.
- This knowledge can interfere with concepts in fraction instruction.
- Instruction often treats the whole number bias as an obstacle to overcome.

**Critical Inconsistencies**

1. Interpreting fraction symbols



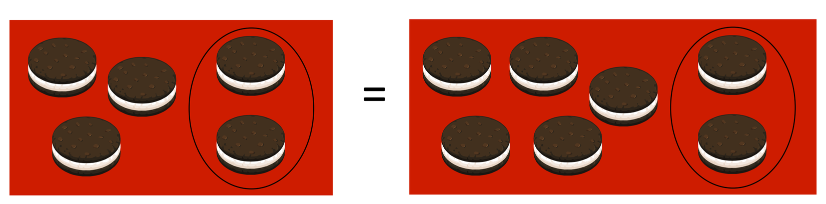
$= \frac{3}{7}$

fractions as two counts (Mack, 1995)

$$\frac{1}{3} < \frac{1}{4}$$

whole number-biased reasoning (Behr et al., 1984)

2. Units: An amount measured against a standard



$$\frac{2}{5} \neq \frac{2}{7}$$

(Mack, 1995; Sophian, 2007)

3. Numerical density

How many numbers are between 1 and 6?

How many numbers are between $\frac{1}{11}$ and $\frac{6}{11}$ ?

(Vamvakoussi & Vosniadou, 2010; Vamvakoussi et al., 2012)

4. Operational patterns

| | Whole Numbers | Fractions |
|---|---|---|
| Addition | *makes bigger* | *makes bigger* |
| Subtraction | *makes smaller* | *makes smaller* |
| Multiplication | **makes bigger** | **makes smaller** |
| Division | **makes smaller** | **makes bigger** |

(Fischbein et al., 1985; Graeber et al., 1989)

**The Whole Number Bias: An Obstacle or Potential for Conceptual Change?**

- Despite these inconsistencies, there are many uniting concepts. For example, many concepts introduced by fractions apply to whole numbers. Operational structures are similar across number type (e.g., multiplicative and additive).
- One goal of mathematics education might be to foster a more generalized conceptual structure to account for both properties of fractions and properties of whole numbers.
- Indeed, the Common Core Standards (2010) recommends drawing on students' whole number knowledge when teaching about fractions, suggesting an integration goal.

**Guiding Questions:**

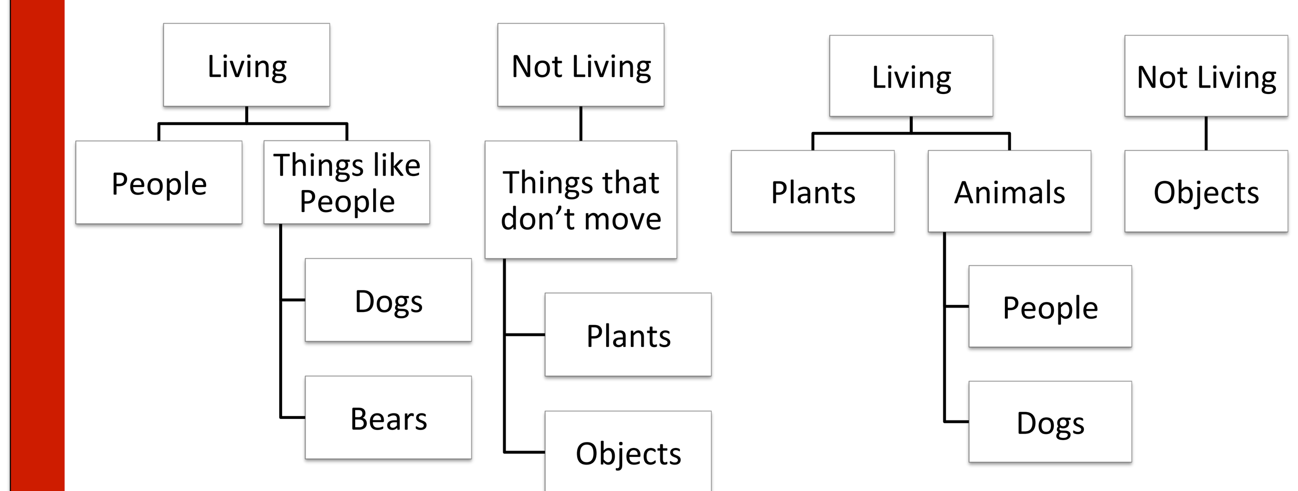
1. Does learning about fractions lead to conceptual change in prior knowledge and foster integration across whole numbers and fractions?
2. How would we know such conceptual change is taking place?
3. When would we expect conceptual change to happen?
4. What are other potential outcomes of learning about fractions?

## Conceptual Change in Science

**What is Conceptual Change?**

There are few theoretical accounts of conceptual change in math, but many in science.

**Carey's *Theory Theory* Approach to Biology**




Children's prior knowledge changes dramatically:

- Individual concepts change (e.g., alive).
- The conceptual structure changes (e.g., reassignment of plants as living).
- Inferences across the structure change.

Due to this reorganization, there is potential for children's thinking about well-understood concepts (e.g., people) to change based on the new structure.

**Vosniadou's *Framework Theory* Approach to the Earth**

| Level | Naïve Theory | Advanced Theory |
|---|---|---|
| **Framework Theory** | **Physical Objects** | **Astronomical Objects** |
| Specific Theory | The earth is flat and gravity pulls down. | The earth is a sphere and gravity pulls towards the center of the earth. |
| Mental Models |  |  |

Children's prior knowledge can change in many ways:

- Learning specific facts can enrich an existing theory.
- Revision at the level of framework theory (in this case, differentiation) is considered *true* conceptual change.
- Under some conditions, the framework theory does not change as new information is added, resulting in synthetic models.

**What Causes Conceptual Change in Science?**

1. Accumulation of facts (e.g., Siegler & Opfer, 2004)
2. Social collaboration (e.g., Vosniadou et al., 2001)
3. Explanation (e.g., Hatano & Inagaki, 1997)
4. Analogical mapping (Carey, 1999)
5. Physical representations (e.g., Vosniadou et al., 2005)


For a reprint of this poster please contact Pooja Sidney at pgupta6@wisc.edu.


## Conceptual Change in Math

**Predictions about Conceptual Change in Math:**

1. Explanatory structure changes over long periods of time.
2. Change can include both differentiation and integration of concepts.
3. Several potential outcomes, or "knowledge states" along the way, depending on the child's specific experiences.

There is little longitudinal evidence. Snapshots of conceptual knowledge at various ages allow us to infer whether or not conceptual change, and thus changes to children's prior whole number knowledge, likely occurs.

**Snapshot: Poorly Integrated Knowledge**

- The whole number bias suggests the assimilation of fraction knowledge into a *framework theory* of whole numbers.
- Assimilation of new facts results in synthetic models of number (Stafyliadou & Vosniadou, 2004).

Changes to original structure: Whole number knowledge structure may be altered, but not reorganized.

**Snapshot: Differentiated Knowledge**

- Verbal reports suggest two separate number systems (Gelman, 1998; Smith et al., 2005).
- Differences in reasoning about numerical density (Vamvakoussi et al., 2011)

Changes to original structure: Whole number knowledge structure may not be altered by new fraction knowledge.

**Snapshot: Integrated Knowledge**

- Steffe's & Olive's (2010) reorganization hypothesis
- Generalized subtraction procedure following a comparison across domains (Peled & Segalis, 2005)

Changes to original structure: Number & arithmetic knowledge may be reorganized, resulting in new insights about whole numbers.

**What Supports Conceptual Change in Math?**

1. Social collaboration (Steffe & Olive, 2010)
2. Supportive representations (TIMA; Steffe & Olive, 2010)
3. Analogical mapping (Sidney & Alibali, 2013)

Typical instructional conditions seem to result in poorly integrated knowledge (the whole number bias).

**Agendas for Future Research**

1. Develop ways of measuring conceptual change. How to measure changes in prior knowledge including integration and differentiation of concepts?
2. What instructional conditions would best support conceptual change in mathematics? The science conceptual change literature provides some predictions about math learning.