Measuring Conceptual Change in Mathematics: Could Learning about Fractions Provoke Changes in Arithmetic Categories?

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Introduction

Fractions: A Catalyst for Conceptual Change

- What children learn about fraction arithmetic is often at odds with what they already know about natural whole number arithmetic (e.g., Ni & Zhou, 2005).

Pattern	Natural Numbers	Fractions
Multiplication makes…	Bigger	Smaller
Division makes…	Smaller	Bigger
Division means…	Grouping	Grouping

- Fractions and whole numbers may seem so incommensurate that young children tend to treat these number representations as indicating separate systems of numbers (e.g., Gelman & Meck, 1992).

Do children progress from viewing fraction and whole number arithmetic as two separate systems of arithmetic to viewing fractions and whole numbers as part of one integrated system, organized by shared concepts?

Measuring Conceptual Change

- Arithmetic problems that are perceived to be more similar may be considered to be in the same category, and problems that are perceived to be different may be considered to be in different categories.

•. As learners gain domain knowledge, the perceived category structure can become reorganized from being organized by surface features to being organized by structural features (e.g., Carey, 1980; Chi, Feltovich, & Glaser, 1981).

Goal 1: To develop measures to capture developmental change in the conceptual organization of arithmetic.

Goal 2: To examine developmental trends in conceptual organization as learners master fraction arithmetic.

Method

Participants

5th grade students (n = 26)

6th grade students (n = 32)

8th grade students (n = 21)

Undergraduates (n = 44)

Measures

Sorting Task (All Age Groups)

Purpose: Assess dimensions along which categories are differentiated (number type or arithmetic operation)

Materials: 16 equations (8 with fractions), presented individually

Instructions: Sort by similarity, more similar equations closer together

Original Presentation

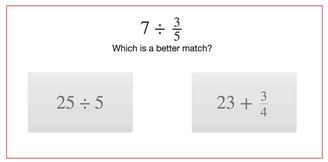
Sorted Primarily by Number Type

Triad Similarity Task (8th graders only)

Purpose: Assess dimensions along which problem similarity is determined (number type or arithmetic operation)

Materials: iPad app, randomly displays 1 of 56 pairs, along with $7 \div \frac{3}{5}$

Instructions: Choose which equation is more similar to $7 \div \frac{3}{5}$



Critical trials (n = 24) pit an operation match (whole number division) against a number type match (fraction addition, subtraction, or multiplication).

Randomly interleaved with 4 kinds of "filler" trials (n = 32).

Results

Measuring Conceptual Change

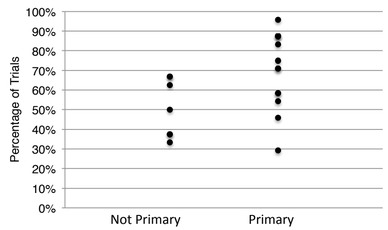
Coding

- Participants' sorts were coded for organization by **operation** or **number type**. Most sorts were organized by both dimensions, one being *primary*.
- Data from the triad similarity task was aggregated within participants: % of critical trials on which they choose the structure-match over the surface-match.

Relationship between the tasks

- On the triad similarity task, 8th grade participants choose a structure-match over a surface-match on an average of 63% of trials.
- Those who sorted primarily by operation were more likely to choose the structure-match (70% of trials) than those who did not sort primarily by operation (51%, $p < .05$, two-tailed *t*-test).

Figure 1: Percentage of Structure-Matches by Sorting Behavior - Operation Primary or Not



Developmental Trends

- There was no clear pattern among 5th graders.
- Most 6th graders sorted primarily by number type.
- Most 8th graders sorted primarily by operation.
- A large majority of adults sorted by operation while ignoring number type all together (Figure 3).

Figure 2: Percentage of Sorts Classified as Primarily by Operation or by Number Type

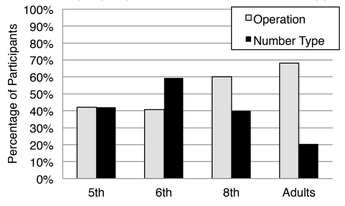
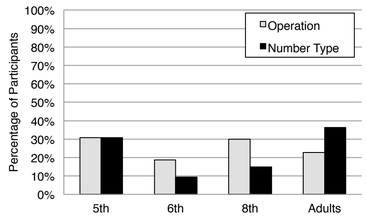


Figure 3: Percentage of Sorts Not Organized by Operation or Number Type, At All



Discussion

Goal 1: Measuring Conceptual Change

- We developed two tasks for measuring conceptual organization of arithmetic categories: a sorting task and a triad similarity task.
- Behavior on these tasks was associated. As expected, participants who attended to operational symbols during sorting were also more likely to attend to operational symbols while equation-matching.

Goal 2: Developmental Trends

- We found the expected developmental trend. Younger students were more likely to prioritize number type (fractions or not) in their arithmetic categories. Older students were more likely to prioritize operation (e.g., division or not).
- This may have implications for transfer and generalization from students' prior knowledge of whole number arithmetic to fraction arithmetic.

For a reprint of this poster, please contact Pooja Sidney at pgupta6@wisc.edu.