

# Developing Operation Sense: Children's and Adults' Arithmetic with Countable and Uncountable Amounts

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## Introduction

What is Operation Sense?

### Early Arithmetic Concepts

#### Number Sense

counting principles,  
relative magnitude,  
absolute magnitude,  
simple calculation

#### Operation Sense

arithmetic principles,  
relationships  
between operations,  
*intuitive models*

(See McIntosh, Reys, & Reys, 1992; Schifter, 1997)

- Both children and adults demonstrate robust arithmetic principle knowledge for well-practiced operations (Dixon, Deet, & Bangert, 2001; Robinson, Ninowski, & Gray, 2005).
- Little is known about children's intuitive models and how they may support connections between symbolic problems and physical actions and objects.

### Measurement of Operation Knowledge

- Children's conceptual models of operations are often measured by having students reason about symbolic equations *only*.
- However, the task context may affect children's performance on these tasks (Prather & Alibali, 2009). In the context of familiar equations, students may draw on their knowledge of number facts.
- As a result, such tasks may not tap operation sense as a more abstract, generalized model of the actions and consequences associated with arithmetic operations.

## Study Goal


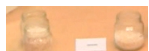


In this study, we examined children's and adults' conceptual representations of arithmetic operations, contrasting tasks with countable objects and tasks with uncountable amounts.

## Method

### Participants

10 First-Graders, 13 Second-Graders, 11 Third-Graders, 7 Fourth-Graders, & 12 Adults

### Tasks

	Countable Math Chips	Uncountable Rice Jars	Correct Strategies
Addition	$9 + 7$		Combining quantities
Subtraction	$13 - 8$		Taking an amount equal to the subtrahend away from the minuend
Multiplication	$4 \times 6$		Iteration of a group or amount to make several equal groups
Division	$15 \div 3$		Dividing the initial quantity into equal groups or amounts

### Countable: Math Chips



### Uncountable: Rice Jar

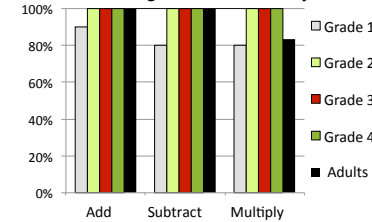


## Results

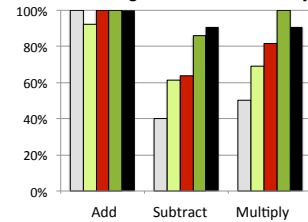
### Addition, Subtraction, and Multiplication Trials

- A Grade (5) x Operation (3) x Context (2) Mixed ANOVA on Correct Strategies:  
Grade,  $F(4, 47) = 4.3, p < .01$ ; Context,  $F(1, 94) = 14.8, p < .01$ ; Context x Operation  $F(2, 94) = 7.5, p < .01$ .
- Addition was easiest for all participants. Success with subtraction and multiplication on uncountable amounts was correlated with grade:  $r = .37, p < .01$  and  $r = .36, p < .01$ , respectively.

### Correct Strategies on Countable Objects

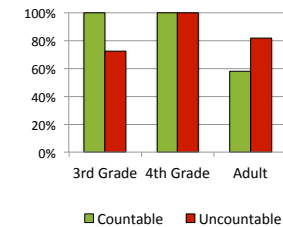


### Correct Strategies on Uncountable Objects



### Division Trials

- Grade (3) x Context (2) Mixed ANOVA :  
Grade x Context  $F(2, 26) = 4.8, p < .05$ .



## Discussion

- By including uncountable amounts, we were able to see a developmental trend in operational knowledge.
- Though elementary students can represent operations with countable objects at an early age, more abstract operational knowledge may develop more slowly.
- In adults, division concepts appear to be less well-developed than other operation concepts.
- This study is a first step towards measuring children's operation sense independently of other number knowledge.

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